

in a remarkable manner Schröter's isolated observation, and renders it probable that in the year 1788 this object presented much the same appearance as at the present day when viewed with a low power eyepiece attached to a small telescope.

I would merely venture to point out that the evidence of Grimaldi's two maps, as published by Riccioli in his *Almagestum Novum*, which is sometimes appealed to as proof of the existence of a large crater at this spot about the middle of the seventeenth century, appears to me to be of very little value indeed. In one of these two maps Linné is represented as a white spot, while Bessel and Sulpicius Gallus appear as craters with shaded interiors. In the other map, which seems to be the more carefully finished production, Linné is also provided with a shaded crater-like interior, in every way comparable with that of Sulpicius Gallus. The force of this evidence in favour of the crater-like form of Linné in 1651 is, however, very much weakened, if not entirely dissipated, by the fact that the peaks of the Apennine range, which in the former map are represented as white spots similar to but smaller than Linné, appear in the latter as a row of small carefully shaded craterlets.

Studies of Mersenius and Copernicus—both of which have been suspected of change—are also included amongst Russell's drawings.

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*Transformation of Hansen's Tables.* By P. H. Cowell, M.A.

Newcomb (*Astron. Papers, Amer. Eph.* vol. i.) has transformed Hansen's theory into longitude and latitude in order to compare it with Delaunay. In this paper I give the transformation of Hansen's tables in order to compare them with the observations. Newcomb's paper has, of course, been of great assistance. It will be seen that my method is to vary Newcomb's results, and thus avoid performing the long calculations that would have been necessary in the absence of his paper.

Hansen's longitude is found from the formula

$$L = f + \Pi + R + R'$$

where  $f$ ,  $R$ ,  $R'$  are functions of  $e$ ,  $I$ ,  $n\delta z$ ,  $s$ , and Hansen's latitude  $\beta$  is found from the formula

$$\sin \beta = \sin I \sin (f + \omega) + s$$

$e$  and  $I$  are arbitrary constants to which Hansen assigns one value in the theory and a different value in the tables.  $n\delta z$  and  $s$  are the symbols under which Hansen calculates the perturbations. As regards these quantities Hansen gives slightly different coefficients in his tables to those that he subsequently

gives in his theory. Moreover in his tables he omits certain terms as not worth tabulating, although in the opening pages of his tables he gives the coefficients that he has calculated, and these coefficients in some cases amount to upwards of a tenth of a second. He has also introduced other modifications such as multiplying all terms in  $n\delta z$  by one factor and all parallactic terms by another factor. Hence it must be clearly understood that throughout this paper "Hansen's tables" refers to what Hansen has tabulated, and not to the preliminary expressions.

The variation between Hansen's tables and Hansen's theory will be denoted by the operator  $\Delta$ .

$$\begin{aligned}\text{Hence} \quad \Delta e &= +0.00000728 \\ \Delta I &= -8''.04\end{aligned}$$

Also  $\Delta n\delta z$ ,  $\Delta s$  denote respectively the variations of  $n\delta z$  and  $s$ . Newcomb has considered the parts of  $n\delta z$  and  $s$  that arise from the action of the Sun only. Hence  $\Delta n\delta z$  and  $\Delta s$  consist of two distinct parts, one part arising from the varied coefficients of arguments due to the action of the Sun, and the other part comprising the planetary and figure of Earth and figure of Moon terms in so far as these terms depend on fresh arguments.

These variations will here be considered separately.

### I. The Variation of the Solar Terms.

Following Newcomb's notation

$$\begin{aligned}f &= (e, g)_0 + (e, g)_1 n\delta z + (e, g)_2 (n\delta z)^2 + \dots \\ R &= -\frac{1}{2} \tan^2 \frac{1}{2} I \sin 2(f + \omega) + \frac{1}{2} \tan^4 \frac{1}{2} I \sin 4(f + \omega) - \dots \\ &= R_1 + R_2 \dots \\ R_1 &= R_{10} + R_{11} n\delta z + R_{12} (n\delta z)^2 + \dots \\ R_2 &= R_{20} + R_{21} n\delta z + R_{22} (n\delta z)^2 + \dots\end{aligned}$$

$R' = sS$  + three periodic terms that require no transformation + nutation that is not considered in this paper; and

$$s = S_0 + S_1 n\delta z + S_2 (n\delta z)^2$$

Hence

$$\Delta L = \Delta(e, g)_0 + \Delta R_{10} + A \Delta(n\delta z) + S_0 \Delta s$$

$$\text{where } A = (e, g)_1 + R_{11} + sS_1 + 2(e, g)_2 n\delta z + \dots$$

$A$  may be reduced to its first term

$$(e, g)_1 = 1 + 0.1098 \cos g + 0.0075 \cos 2g$$

I also copy from Newcomb's paper

$$S_0 = +0.005 \cos \omega - 0.090 \cos (g + \omega) - 0.005 \cos (2g + \omega)$$

Then

$$\begin{aligned} \Delta(e, g)_0 &= +3''.000 \sin g + 0''.206 \sin 2g + 0''.015 \sin 3g \\ \Delta R_{1,0} &= -0''.034 \sin (g + 2\omega) + 0''.359 \sin (2g + 2\omega) \\ &\quad + 0''.033 \sin (3g + 2\omega) \end{aligned}$$

$\Delta(n\delta z)$  and  $\Delta s$  are given in the third columns of the two tables in this paper. The terms in these columns enclosed in brackets are those that do not occur in Hansen's tables; the coefficients are therefore those of the theory with the signs changed. Hence, to form the solar part of  $\Delta L$ , it is only necessary to perform the multiplications and additions indicated by the formula. Adding to  $L$ , as given in the last column of Newcomb's second table,  $L + \Delta L$ , the solar part of the longitude of Hansen's tables is obtained. The result is set down in the second column of the first table.

The following intermediate step of the calculations is given here.

$$\begin{aligned} S_0 \cdot \Delta s &= -0''.288 \sin (2g + 2\omega) + 0''.024 \sin (g + 2\omega) \\ &\quad - 0''.022 \sin (3g + 2\omega) + 0''.018 \sin g \end{aligned}$$

Lastly it must be noted that certain coefficients vary with the time. Hence the value of  $L + \Delta L$  given in Table I. must be supplemented by the terms

$$\begin{aligned} &-0''.182 T \sin (-g - g') - 1''.637 T \sin (-g') - 0''.278 T \sin (g - g') \\ &+ 0''.069 T \sin (g - g' + 2\omega - 2\omega') + 0''.057 \sin (2g - g' + 2\omega - 2\omega') \\ &- 0''.493 T \sin (g - 3g' + 2\omega - 2\omega') \\ &\quad - 0''.388 T \sin (2g - 3g' + 2\omega - 2\omega') \end{aligned}$$

where  $T$  is reckoned in centuries from 1800.0.

For the latitude

$$\begin{aligned} \Delta\beta &= (1 + \frac{1}{2} \sin^2 \beta) \Delta \sin \beta \\ &= (1 + \frac{1}{2} \sin^2 \beta) \left\{ \sin I \sin (f + \omega) \frac{\Delta \sin I}{\sin I} \right. \\ &\quad \left. + \sin I \cos (f + \omega) \Delta f \right. \\ &\quad \left. + \Delta s \right\} \end{aligned}$$

Of the expression within brackets the first term is obtained from the second column of Newcomb's Table IV. by multiplying by the factor  $\frac{\Delta \sin I}{\sin I}$ , or  $-0.0004328$ ; its value is set down in

the fourth column of the second table in this paper; in the second term

$$\begin{aligned}\cos(f+\omega) &= \cos(g+\omega) - 0.1098 \sin(g+\omega) \sin g \\ \Delta f &= \Delta(e, g)_0 + (1 + 0.1098 \cos g) \Delta n \delta z + n \delta z \cos g . 2 \Delta e \\ \sin I \Delta(e, g)_0 &= +0''.269 \sin g + 0''.018 \sin 2g \\ \sin I \Delta n \delta z &= +0''.014 \sin(-g') + 0''.043 \sin(-g-2g') \\ &\quad + 0''.064 \sin(g-2g'+2\omega-2\omega') \\ &\quad \quad \quad + 0''.031 \sin(2g-2g'+2\omega-2\omega') \\ &\quad - 0''.013 \sin(-4g'+2\omega-2\omega') + 0''.014 \sin 2\omega \\ &\quad \quad \quad + 0''.027 \sin(g+2\omega) \\ &\quad + 0''.025 \sin(2g'+2\omega') + 0''.024 \sin(\omega-\omega') \\ &\quad \quad \quad + 0''.049 \sin(g+\omega-\omega') \\ &\quad - 0''.064 \sin(-g'+\omega-\omega') - 0''.395 \sin(g-g'+\omega-\omega')\end{aligned}$$

and  $\sin I n \delta z \cos g . 2 \Delta e$  is less than  $0''.01$ .

The second term of  $\cos(f+\omega)$  is only sensible when taken in connection with the terms

$$+ 0''.269 \sin g - 0''.395 \sin(g-g'+\omega-\omega')$$

and this part of the product is

$$\begin{aligned}&+ 0''.007 \sin(-g+\omega) - 0''.015 \sin(g+\omega) + 0''.007 \sin(3g+\omega) \\ &+ 0''.011 \sin(g-g'+2\omega-\omega') + 0''.011 \sin(g-g'-\omega') \\ &\quad - 0''.011 \sin(-g-g'-\omega') - 0''.011 \sin(3g-g'+2\omega-\omega')\end{aligned}$$

The complete value of the term is given in the fifth column of the second table, the excess of what is there set down over the expression just written representing  $\cos(g+\omega) . \sin I \Delta f$ .

$\Delta s$  is given in the third column

$$\frac{1}{2} \sin^2 \beta . \Delta \sin \beta = -0''.005 \sin(g+\omega) + 0''.002 \sin(3g+3\omega)$$

and  $\beta + \Delta \beta$  is given in the second column of the table,  $\beta$  being taken from Newcomb's fourth table. In both tables only those terms are given which have a coefficient as large as  $0''.1$ , and the results set down must only be considered as accurate to  $0''.02$ .

## II. *The Planetary and other Terms.*

The planetary terms of  $\Delta L = AP + S_0 Q$ , where P, Q are the planetary terms of  $\Delta(n\delta z)$  and  $\Delta s$  respectively.

$$Q = +8''.764 \sin(f + \Pi + 169^\circ 51')$$

The value of P can be taken from Hansen's tables.

$$A = 1 + 0.1098 \cos g + 0.0075 \cos 2g$$

$$S_0 = +0.005 \cos \omega - 0.090 \cos(g+\omega) - 0.005 \cos(2g+\omega)$$

as before.

To this must be added

$$0.022 \cos (g - 2g' + 2\omega - 2\omega') + 0.021 \cos (2g - 2g' + 2\omega - 2\omega')$$

multiplied by certain terms of  $P$ ; for Hansen directs that the sum of the terms in question is to be added to the argument of the evection and to the mean elongation.

The multiplications, so far as  $P$  is concerned, need not be performed, for  $P$  only contains the following short-period terms :

$$0''.256 \sin \Theta \cos g$$

for which  $A$  may be taken as unity.

For the long-period terms the product by the term  $0.1098 \cos g$  in  $A$  is not required for this reason. The observations will be divided into several periods of analysis, and in each period it will be assumed that an error  $a \cos g$  runs through the observations.  $a$  will be found for each period and compared with  $0.1098 P$ .

$$S_0 Q = -0''.394 \{ \sin (2f + \Pi + \omega + 169^\circ 51') + \sin (-\Theta + 169^\circ 51') \}$$

or with sufficient accuracy

$$\begin{aligned} & -0''.394 \{ \sin (2g + \Pi + \omega + 169^\circ 51') + \sin (-\Theta + 169^\circ 51') \} \\ & -0''.088 \cos (2g + \Pi + 169^\circ 51') \sin g \end{aligned}$$

The planetary terms of  $\Delta\beta$  are

$$(1 + \frac{1}{2} \sin^2 \beta) \{ \sin I \cos (f + \omega) P + Q - 1''.00 \}$$

the last term being a term that Hansen adds to all latitudes on account of the figure of the Moon.

Here also, and for a similar reason, it is not necessary to perform the multiplications.

TABLE I.

*Terms in Hansen's Tabular Longitude transformed and in  $\Delta(n\delta z)$ .*

Arg.				Arg.			
$g$	$g'$	$L + \Delta L$	$\Delta(n\delta z)$	$g$	$g'$	$L + \Delta L$	$\Delta(n\delta z)$
1	0	+ 22640.168	...	2	-1	+ 9.719	...
2	0	+ 769.064	...	3	-1	+ 0.670	...
3	0	+ 36.127	...	-1	-2	+ 0.709	-0.475
4	0	+ 1.932	...	0	-2	+ 7.493	-0.014
5	0	+ 0.113	...	1	-2	+ 2.568	-0.025
-3	-1	+ 0.551	...	2	-2	+ 0.192	...
-2	-1	+ 7.666	...	$2\omega - 2\omega'$			
-1	-1	+ 109.953	+0.037	0	0	- 0.266	-0.036
0	-1	+ 670.006	+0.154	1	0	- 2.488	+0.047
1	-1	+ 148.032	+0.024	2	0	- 0.194	...

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Arg.			L+ΔL.	Δ(nδz).	Arg.			L+ΔL.	Δ(nδz).
$\sigma$	$\sigma'$				$\sigma$	$\sigma'$			
-1	-1	+	0.124	-0.053	0	3	-	2.153	...
0	-1	+	2.556	+0.035	-2	2	+	0.425	...
1	-1	-	28.595	-0.036	-1	2	+	6.435	+0.057
2	-1	-	24.452	...	0	2	-	54.980	+0.282
3	-1	-	2.926	...	1	2	-	0.139	+0.021
4	-1	-	0.292	...	2	2	+	0.561	...
-2	-2	+	0.949	...	0	1	+	1.499	-0.055
-1	-2	+	13.200	+0.011	$\omega - \omega'$				
0	-2	+	211.704	+0.010	-1	0	+	0.384	...
1	-2	+	4586.690	+0.717	0	0	+	1.582	+0.266
2	-2	+	2370.137	+0.352	1	0	+	18.167	+0.551
3	-2	+	191.940	...	2	0	+	1.276	+0.011
4	-2	+	14.374	...	-2	-1	-	0.118	...
5	-2	+	1.060	...	-1	-1	-	1.804	-0.020
-1	-3	+	0.453	-0.022	0	-1	-	19.150	-0.711
0	-3	+	8.693	+0.033	1	-1	-	126.459	-4.388
1	-3	+	206.492	+0.060	2	-1	-	8.543	-0.053
2	-3	+	165.556	+0.039	3	-1	-	0.588	...
3	-3	+	14.597	...	0	-2	-	0.153	+0.014
4	-3	+	1.182	...	1	-2	-	0.594	-0.010
0	-4	+	0.134	-0.146	2	-2	-	0.127	...
1	-4	+	7.419	-0.013	$3\omega - 3\omega'$				
$2\omega - 2\omega'$					2	-2	+	0.270	...
2	-4	+	8.125	...	3	-2	+	0.150	...
3	-4	+	0.758	...	1	-3	-	1.247	-0.063
1	-5	+	0.257	...	2	-3	-	3.224	-0.081
2	-5	+	0.344	...	3	-3	+	0.422	+0.027
$2\omega$					2	-4	-	0.226	...
2	1	+	0.416	...	$\omega + \omega'$				
3	1	+	0.265	...	0	1	+	0.112	+0.037
0	0	+	1.253	+0.153	1	1	+	0.571	+0.030
1	0	-	39.281	+0.304	$3\omega - \omega'$				
2	0	-	411.598	-0.012	3	-1	+	0.235	(-0.010)
3	0	-	45.080	...	$\omega - 3\omega'$				
4	0	-	3.997	...	1	-3	-	0.300	+0.018
5	0	-	0.329	...	$4\omega - 4\omega'$				
3	-1	-	0.304	...	2	-3	-	0.356	...
$2\omega'$					3	-3	-	0.640	...
-1	3	+	0.339	-0.063	4	-3	-	0.293	...

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Arg.				Arg.					
$g$	$g'$		$L+\Delta L.$	$\Delta(n\delta z).$	$g$	$g'$	$L+\Delta L.$	$\Delta(n\delta z).$	
1	-4	+	1'177	...	4	-2	-	5'753 (-0'010)	
2	-4	+	30'778	...	5	-2	-	0'991	...
3	-4	+	38'426	...	6	-2	-	0'124	...
4	-4	+	13'900	...	3	-3	-	0'389 (+0'041)	
5	-4	+	1'981	...	4	-3	-	0'384	...
6	-4	+	0'173 (-0'048)		$2\omega-4\omega'$				
2	-5	+	2'746	...	1	-4	+	0'202	-0'021
3	-5	+	4'402	...	$6\omega-6\omega'$				
4	-5	+	1'886	...	3	-6	+	0'292	...
5	-5	+	0'286	...	4	-6	+	0'572	...
2	-6	+	0'156	...	5	-6	+	0'395	...
3	-6	+	0'313	...	$6\omega-4\omega'$				
4	-6	+	0'153	...	4	-4	-	0'150 (+0'016)	
$4\omega-2\omega'$					5	-4	-	0'203	...
2	-2	-	0'534	...	$4\omega$				
3	-2	-	9'370	...	4	0	+	0'422	...

TABLE II.

Terms in Hansen's Tabular Latitude transformed and in  $\Delta s$ ,  
 $\sin(f+\omega) \Delta \sin I$ ,  $\sin I \cos(f+\omega) \Delta f$ .

Arg.		$\beta+\Delta\beta$ .	$\Delta s$ .	$\sin(f+\omega)$ $\Delta \sin I$ .	$\sin I$ $\cos(f+\omega) \Delta f$ .
$g$	$g'$				
-1	1	- 0.322	-0.017	...	...
0	1	- 5.665	...	+ 0.002	...
1	1	- 6.484	...	+ .013	...
2	1	- 5.333	...	+ .003	...
3	1	- 0.640	...	...	...
-2	0	- 1.585	...	...	...
-1	0	- 31.759	...	+ .003	+ 0.007
0	0	- 999.533	- .169	+ .442	- .120
1	0	+ 18461.652	+ 6.405	- 7.981	- .015
2	0	+ 1010.006	+ .136	- .437	+ .134
3	0	+ 61.891	+ .010	- .027	+ .007
4	0	+ 3.977	...	- .002	...
5	0	+ 0.263	...	...	...
-1	-1	+ 0.312	...	...	...
0	-1	+ 5.102	- .022	- .001	...
1	-1	+ 4.867	...	- .013	...
2	-1	+ 6.759	...	- .004	...

Arg.		$\beta + \Delta\beta$ .	$\Delta s$ .	$\frac{\sin(f+\omega)}{\Delta \sin I.}$	$\frac{\sin I}{\cos(f+\omega) \Delta f.}$
$g \quad g'$					
3 -1	+	0.798	...	...	...
2 -2	+	0.142	(+ .026)	...	...
$\omega - 2\omega'$					
0 -1	-	0.797	...	...	...
1 -1	-	12.140	...	...	...
2 -1	-	0.832	...	...	...
-3 -2	+	0.134	...	...	...
-2 -2	+	1.518	...	- .001	...
-1 -2	+	15.570	+ .019	- .009	...
0 -2	+	166.581	+ .037	- .091	+ .032
1 -2	+	623.708	+ .045	- .043	+ .004
2 -2	+	33.368	...	- .001	...
3 -2	+	2.146	...	...	...
4 -2	+	0.145	...	...	...
-1 -3	+	0.655	...	...	...
0 -3	+	7.471	...	- .004	...
1 -3	+	29.733	...	- .003	...
2 -3	+	1.776	...	...	...
3 -3	+	0.121	...	...	...
0 -4	+	0.276	...	...	...
1 -4	+	1.095	...	...	...
$\omega + 2\omega'$					
0 2	+	0.286	...	...	...
1 2	-	2.151	+ .029	+ .002	+ .012
2 2	-	0.326	...	...	...
$3\omega$					
1 0	+	0.108	...	...	...
2 0	-	2.790	...	+ .002	+ .014
3 0	-	6.301	...	...	...
4 0	-	1.021	...	...	...
5 0	-	0.119	...	...	...
$3\omega - 2\omega'$					
2 0	-	0.116	...	...	...
2 -1	-	1.320	...	+ .001	...
3 -1	-	1.276	...	+ .001	...
4 -1	-	0.257	(- .017)	...	...
0 -2	+	0.269	...	...	...



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Arg.	$\beta + \Delta\beta$ .	$\Delta s$ .	$\frac{\sin(f+\omega)}{\Delta \sin I.}$	$\frac{\sin I}{\cos(f+\omega) \Delta f.}$
$g \ g'$				
1 - 2	- 1'622	...	+ '001	...
2 - 2	+ 199'455	+ '027	- '037	+ '032
3 - 2	+ 117'243	+ '020	- '051	+ '016
4 - 2	+ 15'124	(+ '015)	- '006	...
5 - 2	+ 1'518	...	- '001	...
6 - 2	+ 0'140	...	...	...
2 - 3	+ 8'908	...	- '004	...
3 - 3	+ 7'993	...	- '004	...
4 - 3	+ 1'162	(+ '022)	...	...
5 - 3	+ 0'127	...	...	...
2 - 4	+ 0'320	...	...	...
3 - 4	+ 0'390	...	...	...
$3\omega - 4\omega'$				
2 - 3	- 0'153	...	...	...
3 - 3	- 0'103	...	...	...
1 - 4	+ 0'642	+ '017	...	...
2 - 4	+ 6'576	...	...	...
3 - 4	+ 3'679	...	...	...
4 - 4	+ 0'466	...	...	...
2 - 5	+ 0'517	...	...	...
3 - 5	+ 0'410	- '012	...	...
$-\omega'$				
0 0	+ 0'802	(- '015)	...	+ '024
- 2 - 1	- 0'110	...	...	...
- 1 - 1	- 0'467	+ '010	...	- '054
0 - 1	- 4'906	- '020	...	- '198
1 - 1	- 0'606	- '022	...	...
$2\omega - \omega'$				
2 0	+ 0'812	...	...	+ '024
3 0	+ 0'101	...	...	...
1 - 1	+ 0'122	+ '029	...	- '032
2 - 1	- 5'412	+ '037	...	- '198
3 - 1	- 0'672	...	...	- '022
$2\omega - 3\omega'$				
1 - 3	- 0'295	...	...	...
2 - 3	- 0'351	...	...	...
$5\omega - 4\omega'$				

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Arg.		$\beta + \Delta\beta$ .	$\Delta s$ .	$\frac{\sin (f+\omega)}{\Delta \sin I.}$	$\frac{\sin I}{\cos (f+\omega) \Delta f.}$	
<i>g</i>	<i>g'</i>					
3	—4	+	2'419	..	..	
4	—4	+	3'004	...	...	
5	—4	+	1'193	...	...	
6	—4	+	0'214	...	...	
3	—5	+	0'218	...	...	
4	—5	+	0'346	...	...	
5	—5	+	0'161	...	...	
4 $\omega$ —3 $\omega'$						
3	—3	—	0'208	...	...	
5 $\omega$ —2 $\omega'$						
4	—2	—	0'246	...	...	
5	—2	—	0'145	...	...	

*A Spectrographic Study of  $\beta$  Lyræ.* By Rev. Walter Sidgreaves, S.J., Stonyhurst College Observatory.

*Introduction.*—The more the spectrum of  $\beta$  Lyræ is studied, the more evident it becomes that its photographs cannot be too greatly multiplied ; and one of the objects of this paper is to show that a suitable telespectrograph would be well employed if devoted solely to this purpose until a series of plates had been collected containing many for each day of the light-period.

The attempt made here during the last two years has been greatly frustrated by the extraordinarily cloudy state of the nights. And the result, so far, is disappointing, inasmuch as many more photographs are wanted, in order to support or undermine the constructive explanations suggested during a rather lengthy study of the plates already collected.

All the photographs have been taken with the small prismatic camera of 4 inches aperture described in the December number of the *Monthly Notices*, 1901, in the introductory part of the paper on *Nova Persei*. They cover the spectrum from H $\beta$  to the head of the hydrogen series, and on one of the stronger photographs the lines can be counted up to H $\pi$ . But H $\beta$  and the lines beyond H $\eta$  are too weak for any trustworthy comments, and the enlargements have been cut down accordingly.

Professor Vogel, discussing the Potsdam photographs of